MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2015/2016

TSE 2351 – INTRODUCTION TO FORMAL METHODS

(All sections / Groups)

4 MARCH 2016
3.00 p.m – 5.00 p.m
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This Question paper consists of 5 pages with 5 Questions only, excluding the cover page.

2. Attempt ALL **FIVE** out of **FIVE** questions. All questions carry equal marks and the distribution of the marks for each question is given.

3. Please print all your answers in the Answer Booklet provided.

Question 1 (3 + 4 + 3 marks)

a. Examine the two relations below:

$$
\begin{aligned}
register = \{ \quad & ali \mapsto tgd3351, \\
& bob \mapsto tpt3101, \\
& claire \mapsto tdb2121, \\
& david \mapsto tpt3101 \\
\}
\end{aligned}
$$

$$
\begin{aligned}
subject = \{ \quad & tdb2121 \mapsto database, \\
& tma1101 \mapsto calculus, \\
& tgd3351 \mapsto game, \\
& tpt3101 \mapsto project \\
\}
\end{aligned}
$$

What are the elements of the relational composition *register ; subject*.

b. You are given two schemas M and P as defined below. What is $M \vee P$? Show all the necessary steps.

$$
\begin{array}{|l|}
\hline
M \triangleq [\\
\quad n : 3..100 \\
\quad U : \mathbb{P}\, \mathbb{N} \\
\quad | \\
\quad n \in U \\
] \\
\hline
\end{array}
$$

$$
\begin{array}{|l|}
\hline
P \triangleq [\\
\quad n : \mathbb{Z} \\
\quad U : \mathbb{P}\, \mathbb{Z} \\
\quad | \\
\quad n < 50 \\
] \\
\hline
\end{array}
$$

c. State THREE advantages of a formal specification language (for example Z) as compared to an informal one (for example UML).

Continued .....

Question 2 (4 + 3 + 3 marks)

a. You are given two Z schemas X and Y as defined below. Construct $X \gg Y$. Show all the necessary steps.

$$
\begin{array}{|l}
X \triangleq [\\
\quad n?, s, s!, y! : \mathbb{N} \\
\quad | \\
\quad s! = s + n? \ ; \\
\quad y! = n? * 9 \\
\quad]
\end{array}
$$

$$
\begin{array}{|l}
Y \triangleq [\\
\quad y?, b, b', s? : \mathbb{N} \\
\quad | \\
\quad y? = b' - s? \ ; \\
\quad b > s? + y? \\
\quad]
\end{array}
$$

b. Three sequences, X, Y and Z are defined as follows:

$X = \langle 10,20,30 \rangle$
$Y = \langle 66,77,88,20,30 \rangle$
$Z = \langle 101,102,103 \rangle$

Find the following:

i. ran $(X \frown Z)$

ii. last (rev Y) $\frown$ head(tail Z)

iii. $Y \rhd X$

c. Assume that you have a set that stores the price of items a customer bought in a store. The set is defined as:

price = $\mathbb{P}\ \mathbb{Q}$

Note: $\mathbb{Q}$ is notation for rational number

The set has the following structure:

price = {32.50, 449.9, 132.78, ...}

Specify a function (in Z) that returns the total amount the customer has to pay for the items bought.

Continued .....

Question 3 (2 + 5 + 3 marks)

Assume that you are asked to write a formal specification (using Z notation) for a system that keeps track of book rental. The state schema is given below:

$$
\begin{aligned}
BookRental \triangleq [\ & customer: \mathbb{P}\ Person \\
& bookCollection: \text{bag}\ Book \\
& bookBorrowed: \text{bag}\ Book \\
& bookAvailable: \text{bag}\ Book \\
& bookOnLoan: Person \leftrightarrow Book \\
| \\
& bookBorrowed \uplus bookAvailable = bookCollection \\
& \text{dom}\ bookOnLoan \subseteq customer \\
]
\end{aligned}
$$

The following are descriptions of the state variables:

State variable	Description
customer	The set of known customers to the library
bookCollection	The bag of books collection. Eg: {Geisha ↦ 6, The Help ↦ 5}
bookAvailable	The bag of books available. Eg: {Geisha ↦ 4, The Help ↦ 3}
bookRented	The bag of books rented. Eg: {Geisha ↦ 2, The Help ↦ 2}
bookOnLoan	The relation to keep track of books borrowed by users. Eg: {Ali ↦ Geisha, Ming ↦ Geisha}

a. Specify (in Z), an operation schema called *NewCustomer*. This operation specifies actions to be taken to save the name of a new customer. The operation has the following signature:
   Input : customer name

b. Specify (in Z), an operation schema called *BorrowBook*. This operation specifies actions to be taken when a book is borrowed. The operation has the following signature:
   Input : book title and customer name

c. Specify (in Z), an operation schema called *SearchCustomer*. This operation specifies actions to be taken when a query is made to search for books borrowed by a customer. The operation has the following signature:
   Input : customer name
   Output : a set of book titles rented by the customer

Continued .....

Question 4 (3 + 4 + 3 marks)

a. The following state schema *BirthdayBook* records people's birthdays.

$$
\begin{aligned}
BirthdayBook \triangleq\ & [\ known: \mathbb{P}\ Name \\
& \quad birthday: Name \rightarrow Date \\
& | \\
& \quad known = \text{dom}\ birthday \\
&]
\end{aligned}
$$

One of the operation schema, *AddBirthday*, is to add a new birthday.

$$
\begin{aligned}
AddBirthday \triangleq\ & [\ \Delta\ BirthdayBook \\
& \quad name?: Name \\
& \quad date?: Date \\
& | \\
& \quad name? \notin known \\
& \quad birthday' = birthday \cup \{name? \mapsto date?\} \\
& \quad known' = known \cup \{name?\} \\
&]
\end{aligned}
$$

Define a schema which describes the conditions when the error '$name? \notin known$' occurs and specify the appropriate report to produce.

b. Specify a function (in Z notation) that takes as an input a sequence of natural numbers and returns the sum of numbers greater than 1 and less than 20 ($1 < x < 20$).

c. Using a suitable example, describe the difference between a ***sequence*** and a ***bag***.

Continued .....

Question 5 (5 + 3 + 2 marks)

a. In order to model set intersection ($\cap$) on two sequences, a function, ***intersect***, is defined as follows:

$[X]$

$\text{intersect}: \text{seq}\, X \times \text{seq}\, X \rightarrow \text{seq}\, X$

$$\begin{aligned}
&\forall x: X;\ s, t: \text{seq}\, X \bullet \\
&\quad (\text{intersect}\ (\langle\rangle, t) = \langle\rangle\ ) \land \\
&\quad (x \in \text{ran}\ t \Longrightarrow \\
&\qquad \text{intersect}\ (\langle x\rangle \frown s, t) = \langle x\rangle \frown \text{intersect}\ (s, t)) \land \\
&\quad (x \notin \text{ran}\ t \Longrightarrow \\
&\qquad \text{intersect}\ (\langle x\rangle \frown s, t) = \text{intersect}\ (s, t))
\end{aligned}$$

Prove by sequence induction that the following statement is true.

ret(inter(s,t)) = ret t $\cap$ ret s

You have to prove both the base case and the inductive step.

Note: The inductive hypotheses is: ret(inter($\langle x\rangle \frown s$, t)) = ret ($\langle x\rangle \frown s$) $\cap$ ret t

b. Determine if the following Hoare triples are true or false. Substantiate your answer.

i. $\forall a, x: \mathbb{Z} \bullet \{a=0\}$ while x>a do x:=x-1 $\{ x=0\}$

ii. $\forall i, j, m: \mathbb{Z} \bullet \{i \neq j\}$ i>j then m:=i-j else m:=j-i $\{m > 0\}$

iii. $\forall i, j, m: \mathbb{Z} \bullet \{i = 3 * j\}$ i>j then m:=i-j else m:=j-i $\{m - 2 * j = 0\}$

c. Briefly describe the FIVE steps in the process of developing and using formal specification.

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